1 Cholesky Factorization

Suppose $A = LL^T$ where L is lower triangular. Then

$$A = \begin{bmatrix} A_{11} & A_{21} & \dots & \\ A_{21} & A_{22} & & \\ \vdots & & \ddots & \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & \dots & 0 \\ L_{21} & L_{22} & & \\ \vdots & & \ddots & \\ L_{n1} & L_{n2} & \dots & L_{nn} \end{bmatrix} \begin{bmatrix} L_{11} & 0 & \dots & 0 \\ L_{21} & L_{22} & & \\ \vdots & & \ddots & \\ L_{n1} & L_{n2} & \dots & L_{nn} \end{bmatrix}^T$$

$$= \begin{bmatrix} L_{11} & 0 & \dots & 0 \\ L_{21} & L_{22} & & \\ \vdots & \vdots & \ddots & \\ L_{n1} & L_{n2} & \dots & L_{nn} \end{bmatrix} \begin{bmatrix} L_{11} & L_{21} & \dots & L_{n1} \\ 0 & L_{22} & \dots & L_{n2} \\ \vdots & & \ddots & \\ 0 & 0 & \dots & L_{nn} \end{bmatrix}$$

and so for $j \geq k$ $A_{jk} = \sum_{i=1}^{k} L_{ji} L_{ki}$. If the system is solvable then for $k = 1..n$

$$L_{kk} = \sqrt{A_{kk} - \sum_{i=1}^{k-1} L_{ki} L_{i1}}$$

$$L_{jk} = \frac{A_{jk} - \sum_{i=1}^{k-1} L_{ji} L_{ki}}{L_{kk}}$$

where $j > k$.This permits an algorithm

```
function L = cholesky(a)

n = length(a);
L = zeros(n,n);

for k = 1:n
  if (k == 1) d = 0; else; d = sum(L(k,1:k-1).*L(k,1:k-1)); endif
  L(k,k) = sqrt(a(k,k) - d);
  for j = k+1:n
    L(j,k) = (a(k,j) - sum(L(k,1:j-1).*L(j,1:j-1)))/L(k,k)
  endfor
endfor

endfunction
```

which has run-time complexity $O(n^3)$.